GEOMETRIC MODEL IDENTIFICATION OF A SERIAL ROBOT

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Robots find their applications in various fields and are used to perform repetitive and adaptive tasks very accurately. This requires exact kinematic parameters of the robot. Generally, for a serial robot, these parameters are represented using the Denavit-Hartenberg (DH) parameters, whose nominal values are provided by the robot manufacturers. In this paper, a technique is proposed to determine the exact DH parameters of a serial robot. For this, each joint of the robot is rotated while the others are locked. Hence, the end-effector moves in a circle, which can be measured using external measurement devices, say a theodolite, a vision system or a laser scanner. From these measurements, the axis and center of the circles traced by the points are determined. If the joint axes are represented using dual vectors, the exact DH parameters can be extracted with the help of Dual Vector Algebra proposed here. The proposed technique has an advantage that it does not require any calibration of base frame of the robot with the measurement frame. Since, the technique allows one to determine the exact DH parameters at the site of installed robot, the robot need not to be taken to a separate calibration section. The proposed measurement methodology is simulated in a CAD software environment using the CAD model of a KUKA KR5 robot.

1. INTRODUCTION

Robots are used extensively in industries to perform various tasks, such as pick and place operation, welding, painting, insertion of components, etc. The robots were earlier used in teach-mode where several points to reach were taught by a trained personnel and the robots would repeatedly perform the tasks. As the robots typically have very high repeatability, these tasks can be performed satisfactorily. Now-a-days, robots also support program-mode in which the points and paths are defined through programs. The robots, however, have poorer accuracy, as compared to their repeatability and hence the tasks performed using a program may not be very accurate. So, there exists a need to improve the accuracy of the robot under study.

The geometric model of a serial robot relates the transformations between the joint-space and the task-space. Note that the robots are generally controlled in the joint-space whereas the tasks are usually defined in the Cartesian space. Denavit-Hartenberg (DH) parameters [1] are generally used for the geometric representation of a serial robot. The DH parameters of an industrial robot are usually provided by the robot manufacturers either in the form of specifications or through engineering drawings. These parameters may not be the same in an actual robot due to manufacturing and assembling errors in components, fatigue of components or their wear and tear. As a result, if a robot is commanded to go to a configuration *A*, it would go to another configuration *B*. In order to improve its accuracy, identification of exact kinematic parameters and robot calibration are required. In the former, the exact kinematic parameters are determined by performing certain experiments and the measured parameters are used in the robot controller instead of the nominal parameters. If the controller cannot be modified, the determined parameters are used as nominal parameters in the Robot Calibration. An overview on Robot Calibration is given in Roth *et al.* [2]. In this paper, identification of exact kinematic parameters, particularly the DH parameters, is discussed.

An analytical method to extract the DH parameters was proposed by Barker [3] using Vector Algebra. Though the method was simpler, no experimental validation was reported. A generic method was proposed by Hayati and Mirmirani [4] to estimate the DH parameters through a linear kinematic model which relates the error in link parameters with the end-effector positioning error. An extra parameter β was added for parallel or near-parallel consecutive joint axes to avoid numerical instabilities during estimation. Another experimental method was proposed by Abderrahim and Whittaker [5], extending Stone's model [6], and using Paul's back-multiplication technique. Rousseau *et al.* [7] identified the DH parameters automatically using machine vision system by extending the method proposed by Hayati and Mirmirani [4], to closed kinematic chains. They proposed solving a system of nonlinear equations in order to determine the unknown parameters. All the reported work, except by Barker [3], used numerical techniques to solve the equations. In this paper, an identification technique is proposed to determine the exact DH parameters of a serial robot. It extends the analytical methodology developed by Rajeevlochana *et al.* [8] to determine the exact DH parameters using the Dual Vector Algebra and Plücker coordinates representation of lines in 3D space. Compared to Barker [3], the methodology has a very compact and elegant representation. The proposed measurements do not require base transformation of the measurement device, i.e., the calibration of measurement frame with respect to the base frame. Thus, it can be used efficiently in an industrial setup without disrupting much of the production process. As a proof of concept, measurements were performed in a simulated environment using a CAD assembly of an industrial robot namely KUKA KR5, and the results obtained matched closely with the given specifications.

2. OVERVIEW

In this section, the mathematical formulations used in the identification of actual DH parameters are discussed.

2.1 Geometric model

The DH parameters are generally used for the geometric modeling of a serial robot shown in Figure 1(a) and defined in Table 1. A set of four DH parameters relate the coordinate frames attached to two consecutive links of a robot are shown in Figure 1(b), where Frame (i+1) is attached to Link i, and Frame i is attached to Link (i-1). Detailed explanation and the rules of attaching the frames are given in Saha [9].
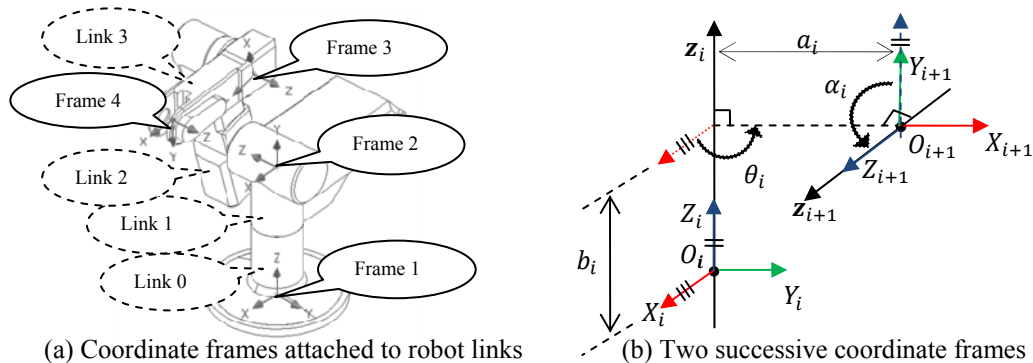

(a) Coordinate frames attached to robot links   (b) Two successive coordinate frames

Figure 1.   Representation of DH parameters

Table 1. Definition of DH parameters

Parameter	Description
Joint Offset (b_i)	Distance between X_i and X_{i+1} along Z_i
Joint Angle ($\theta_{i)}$)	Angle between X_i and X_{i+1} about Z_i
Link Length (a_i)	Distance between Z_i and Z_{i+1} along X_{i+1}
Twist Angle (α_i)	Angle between Z_i and Z_{i+1} about X_{i+1}

2.2 Dual Vector Algebra

A line in 3D space can be represented using a position vector $\mathbf{p}$ to a point P on the line and a free vector $\mathbf{a}$ as the direction of the line, as shown in Figure 2(a). Alternatively, one can use the concept of dual vector consisting of a real part and a dual part to relate the above vectors $\mathbf{p}$ and $\mathbf{a}$ to represent the same line in 3D space. The dual part has the dual entity ε, which is nilpotent, i.e., $\varepsilon^2 = 0$ [10]. A dual vector $\hat{\mathbf{a}}$ is then expressed as

$$\hat{\mathbf{a}} = \mathbf{a} + \varepsilon \mathbf{a}^* = \begin{bmatrix} a_x \, a_y \, a_z \end{bmatrix}^{\mathrm{T}} + \varepsilon \begin{bmatrix} a_x^* \, a_y^* \, a_z^* \end{bmatrix}^{\mathrm{T}} \tag{1}$$

where $\mathbf{a}$ is called the resultant vector and $\mathbf{a}^*$ is called the moment vector, which are related as

$$\mathbf{a}^* = \mathbf{p} \times \mathbf{a} \tag{2}$$

The six scalar quantities of Eq. (1), i.e., $a_x, a_y, a_z, a_x^*, a_y^*,$ and a_z^* are nothing but the Plücker coordinates of a line. Dual Vector Algebra is often used in the field of displacement analysis, kinematic synthesis and dynamic analysis of spatial mechanisms as it produces concise and compact notations as reported in [11-12]. Operations on two dual vectors, say $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_2$ shown in Figure 2(b) are performed similar to those on three dimension vectors. The dot- and cross-products of two dual vectors, also referred as line dot-and cross-products in Fischer [10], are defined by

$$\hat{\mathbf{a}}_1 . \hat{\mathbf{a}}_2 = (\mathbf{a}_1 + \varepsilon \mathbf{a}_1^*) . (\mathbf{a}_2 + \varepsilon \mathbf{a}_2^*) = \mathbf{a}_1 . \mathbf{a}_2 + \varepsilon (\mathbf{a}_1 . \mathbf{a}_2^* + \mathbf{a}_1^* . \mathbf{a}_2) \tag{3}$$

$$\hat{\mathbf{a}}_1 \times \hat{\mathbf{a}}_2 = (\mathbf{a}_1 + \varepsilon \mathbf{a}_1^*) \times (\mathbf{a}_2 + \varepsilon \mathbf{a}_2^*) = \mathbf{a}_1 \times \mathbf{a}_2 + \varepsilon (\mathbf{a}_1 \times \mathbf{a}_2^* + \mathbf{a}_1^* \times \mathbf{a}_2) \tag{4}$$

Note that the resultant of the dot-product of dual vectors is a dual number, consisting of two scalar values, instead of two vectors as in the case of a dual vector.

(a) Line in 3D space (b) Two lines (c) Intersection of plane and line
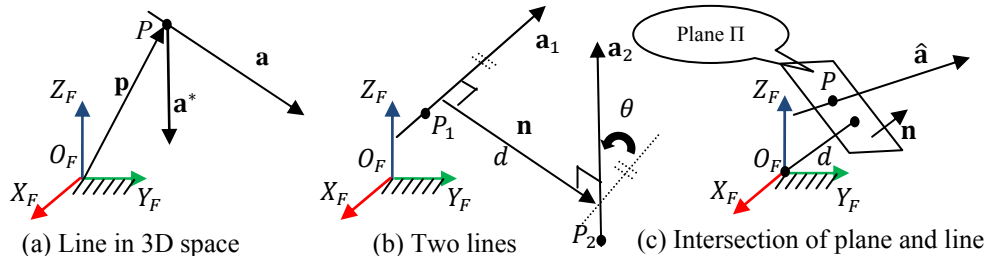
Figure 2. Representation of lines and plane using dual vectors and Plücker coordinates

The dot-product and cross-product between two dual vectors can be used to determine if two lines in 3D space, represented using dual vectors are parallel, intersecting or skewed as shown by Ketchel and Larochelle [13]. This concept was used to determine the DH parameters of a serial robot. Note here that the equation of a plane in generalized form is given by Eq. (5) in which α, β and γ are the components of the normal vector $\mathbf{n}$ to the plane and d is the scalar distance between the plane and the origin.

$$\alpha x + \beta y + \gamma z + d = 0 \tag{5}$$

where, $\alpha, \beta,$ and γ form the normal vector $\mathbf{n} = [\alpha \ \beta \ \gamma]^{\mathrm{T}}$

Moreover, the plane Π can be represented using the Plücker coordinates as in Eq. (6).

$$\Pi = [\mathbf{n} : d] \tag{6}$$

Intersection of a line, represented using a dual vector $\hat{\mathbf{a}}$, and plane Π results in a point as shown in Figure 2(c), provided the line is not parallel to the plane. The expression for the point of intersection $\mathbf{p}$ is compact and given in [14] as

$$\mathbf{p} = \frac{\mathbf{n} \times \mathbf{a}^* + d\mathbf{a}}{\mathbf{a}.\mathbf{n}} \tag{7}$$

2.3 Circle Fitting in 3D Space

A circle in the 3D space can be fitted from the data of the points not lying exactly in a plane due to measurement errors. The methodology explained in this section is to determine the coordinates of the center of the circle and the direction of the normal to the plane containing the circle. To fit a circle, the following steps are to be undertaken.

A set of m data points, say, (x_i, y_i, z_i for i=1, 2, …,m), is to be measured using a technique explained in Section 3.2. This is used to form a matrix of data points $\mathbf{M}$ given in Eq. (8). The matrix $\mathbf{M}$ is then transformed with respect to the centroid of the data points obtained by taking the mean of m data points denoted by $(\overline{x}, \overline{y}, \overline{z})$. This forms a matrix $\mathbf{N}$ given in Eq. (8)

$$\mathbf{M} \equiv \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_m & y_m & z_m \end{bmatrix}_{m \times 3} ; \quad \mathbf{N} \equiv \begin{bmatrix} x_1 - \overline{x} & y_1 - \overline{y} & z_1 - \overline{z} \\ x_2 - \overline{x} & y_2 - \overline{y} & z_2 - \overline{z} \\ \vdots & \vdots & \vdots \\ x_m - \overline{x} & y_m - \overline{y} & z_m - \overline{z} \end{bmatrix}_{m \times 3} \tag{8}$$

The singular value decomposition (SVD) of matrix $\mathbf{N}$ will give two orthogonal matrices $\mathbf{U}$ and $\mathbf{V}$ and one diagonal matrix containing positive singular values of $\mathbf{S}$, i.e.,

$$\mathrm{SVD}(\mathbf{N}) = \mathbf{U}_{m \times m} \mathbf{S}_{m \times 3} \mathbf{V}_{3 \times 3}^{\mathrm{T}} \tag{9}$$

The first two columns of $\mathbf{V} \equiv (\mathbf{v_1} \ \mathbf{v_2} \ \mathbf{v_3})$, namely $\mathbf{v}_1$ and $\mathbf{v}_2$, give an orthonormal basis for plane Π. The third column $\mathbf{v}_3$ corresponding to the lowest singular value gives normal $\mathbf{n}$. The data

points are then transformed to the two-dimensional plane spanned by orthogonal basis $\mathbf{v}_1$ and $\mathbf{v}_2$. As reported by Shakarji [15], a least square fit of a circle to the (x, y) pairs will give the center (c_1, c_2) and radius r. The equation of the circle is given by

$$(x - c_1)^2 + (y - c_2)^2 = r^2 \text{ or, } x^2 + y^2 = 2xc_1 + 2yc_2 + k_3 \tag{10}$$

where $k_3 \equiv r^2 - c_1^2 - c_2^2$. The coordinates of the center (c_1, c_2) are then mapped back into the space of original data by taking the inner product of coordinates of the center to the plane Π. Further, the coordinates of the centroid are added to obtain the actual center C as $[c_x, c_y, c_z]^T$.

3. IDENTIFICATION OF THE DH PARAMETERS

The nominal DH parameters of serial robots are usually provided by the robot manufacturer either in the form of specifications or they can be determined from the diagram showing the dimensions and reach of the robot. The exact parameters however, can vary slightly from the nominal values and a need exists to determine those.

3.1 Methodology

An analytical methodology was proposed by Rajeevlochana *et al.* [8] to determine the exact DH parameters of a robot. For the sake of completeness, certain parts of it are presented in this section. For the Joint i of a serial robot, the unit vector parallel to its axes denoted with $\mathbf{z}_i$ and a point on the axis $\mathbf{p}_i$ are measured in a frame attached to the base-link, i.e., Link 0. These joint axes shown in Figure 3(a) are then represented as dual vectors.

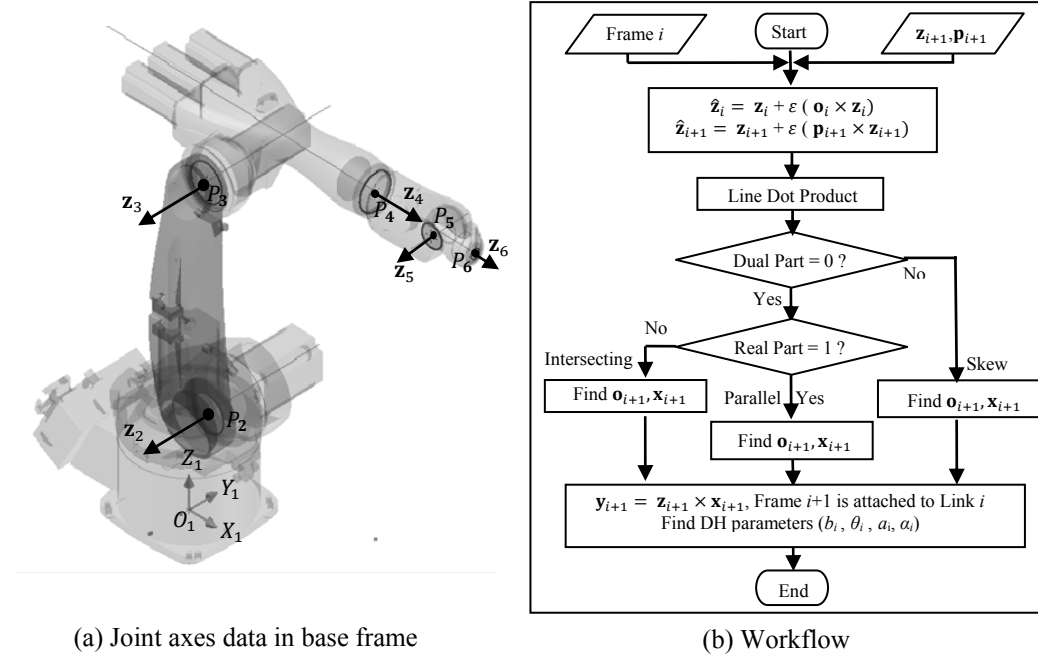| (a) Joint axes data in base frame	(b) Workflow

Figure 3. Methodology to extract DH parameters (Rajeevlochana *et al.*, [8])

From the first joint, the consecutive joint axes are checked if they are parallel, intersecting or skewed with each other. At each iterative step i, the DH Frame i attached to Link (i-1) is known. The dual vector $\hat{\mathbf{z}}_{i+1}$ corresponding to axis of Joint (i+1) is computed. Based on the type of relationship between joint axes, the origin O_{i+1} of the next DH frame, i.e., Frame (i+1) is determined as intersection of lines. The common normal between the joint axes is then determined, which is the X axis of the next DH frame i.e., X_{i+1}. The axis Y_{i+1} is determined using the right-hand rule, thus Frame (i+1) is completely determined. The corresponding DH parameters are determined using vector algebra and trigonometric functions. A flowchart of the methodology is shown in Figure 3(b).

3.2 Measurement Technique

A measurement technique is proposed to determine the exact DH parameters of an actual robot. A fixture is to be attached on the last link, i.e., the end-effector, of the robot. A measurement device is to be placed at any arbitrary location, known as Measurement Frame M. From the first joint to the last, each joint is rotated, locking all others. The coordinates of the point E on fixture is measured in Frame M. Note that the measured points lie on a circle in 3D space. An example for rotation of Joint 1 is shown in Figure 4(a). To determine the direction of normal of the plane containing the circle and its center, actually three measured points are sufficient for the corresponding joint. However, to have improved accuracy, the measurement of the point E is done for a larger number of points. The center and normal of the circles traced by rotation of each joint are determined using the Least Square Fit method as explained in Section 2.3. From several measured points on the base plane of the robot, the equation of base plane can be determined.

Note here that the center of the circle corresponding to the motion of Joint 1 as in Figure 4(b) need not be at the bottom of the base-link. The point of intersection of the axis of Joint 1 i.e., $\hat{\mathbf{z}}_1$, with the base plane Π is determined as O_1 using Eq. (7). This is to be considered as the origin of Frame 1 denoted as O_1.
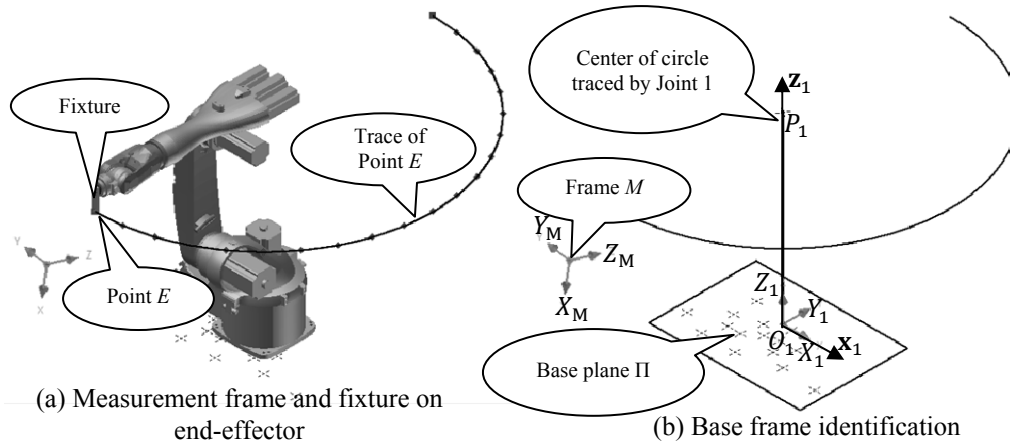

Figure 4. Proposed measurement technique for identification of DH parameters

(a) Measurement frame and fixture on end-effector

(b) Base frame identification

The Z axis of the Frame 1, i.e., Z_1 is same as the axis of Joint 1. By definition, X_1 axis can be assigned arbitrarily, such that it remains perpendicular to Z_1. For example, it can be assigned as the vector from O_1 to any point measured on the base plane Π.

Axis Y_1 can then be determined using the right-hand rule so that the Frame 1 is completely known. The exact DH parameters of the robot under study can be obtained using the methodology explained in Section 3.1. Note here that Frame 1 and all dual vectors representing the joint axes are known in Frame M. Hence, no calibration with respect to the base frame is required, as necessary in some of the identification technique.

4. EVALUATION OF THE IDENTIFICATION METHOD

The proposed measurement technique was tested in a simulated environment. The CAD model of a KUKA KR5 industrial robot was imported inside Autodesk Inventor CAD software and a measurement Frame M was arbitrarily placed in the assembly environment. An Autodesk Inventor addin was developed using Visual C# to rotate the joints of the robot, one at a time. As the robot joints moved, the coordinates of the point on the end-effector fixture were measured in Frame M, as shown in Figure 5(a). Note that the point on the end-effector fixture should not be along the axis of Joint 6. Singular Value Decomposition and Least Square Fit operations were performed on these points using MATLAB to determine the position of joint axes. The exact DH parameters were then determined using the proposed methodology of Rajeevlochana *et al.*, [8] which is outlined in Figure 3(b). For the extraction, the identified DH frames are attached as shown in Figure 5(b).

4.1 Simulation Results and Discussion

To simulate the measurement process, two sets of random errors, having values of ± 0.01 mm and ± 0.1 mm were induced with the measured data. The center and axis of the circles were determined and then the DH parameters were extracted as given in Table 2, along with the parameters obtained from the specifications of the robot.

(a) Input as points measured in Frame M (b) DH Parameters extracted
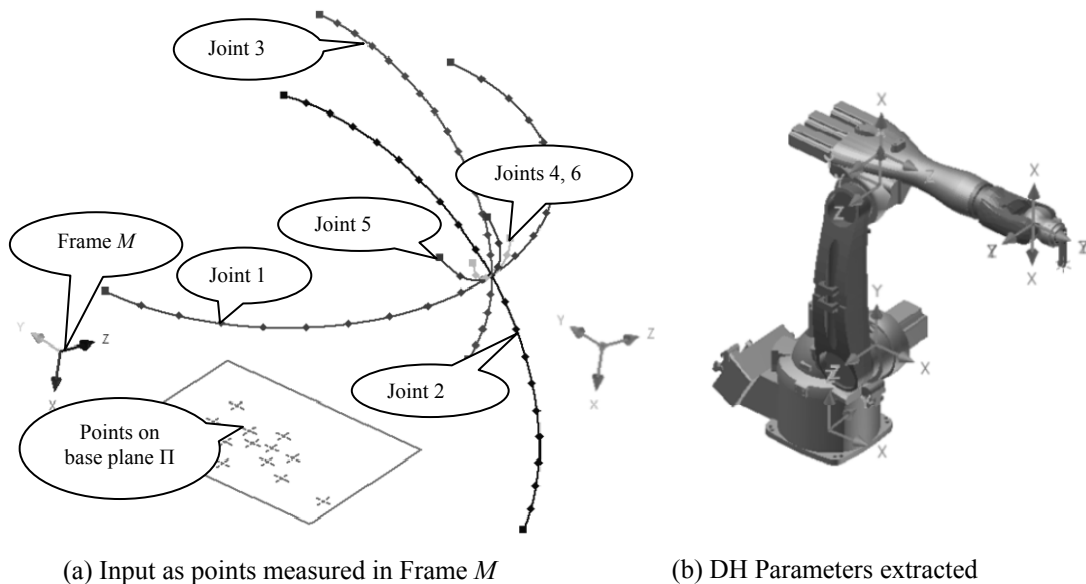
Figure 5. Identification of DH parameters of KUKA KR5 industrial robot

Table 2. Measured DH parameters and the specifications of a KUKA KR5 industrial robot

Joint No.	Joint Offset (b) (mm)			Link Length (a) (mm)			Twist Angle (α) (degree)		
	Specif-ication	Measured (with error)		Specif-ication	Measured (with error)		Specif-icaiton	Measured (with error)	
		± 0.01 mm	± 0.1 mm		± 0.01 mm	± 0.1 mm		± 0.01 mm	± 0.1 mm
1	400	399.985	399.955	180	179.983	180.099	90	89.998	90.002
2	0	0	0	600	600.006	599.992	0	0	0
3	0	0.027	0.092	120	120.044	119.984	90	89.996	90.005
4	620	620.036	620.144	0	0	0	90	89.996	90.023
5	0	-0.131	-0.1	0	0	0	90	89.997	90.022
6	0	0	0	0	0	0	0	0	0

Note that as the joint angles are variables, they are not included in the comparison. The results are in close match with the specifications even though the errors were taken as high as ± 0.1 mm. Hence, the proposed methodology is suitable to be used with not so accurate measurement devices. The simulation of the measurement technique was required to know the range of accuracy of the measurement device required to perform actual measurement of the coordinates of the end-effector point. Accordingly, in future, measurements will be made from a suitable measuring device to determine the actual DH parameters of a KUKA KR5 industrial robot.

5. CONCLUSION

A robust measurement technique is proposed to determine the exact DH parameters of a serial robot. It is an application of the methodology proposed by Rajeevlochana *et al.*, [8] which uses the concept of Dual Vector Algebra. A point on the end-effector of the robot is measured from an arbitrary measurement frame, when the joints of the robot are rotated one at a time, locking all others. The results were generated using simulation on the model of KUKA KR5 robot developed in Autodesk Inventor. The proposed measurement technique can be used to determine the exact DH parameters of an actual robot using a measurement device which has an accuracy of 0.1 mm or better.

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